

$$\begin{aligned}
T_{--} &= \partial_- \mathbf{X} \cdot \partial_- \mathbf{X} \\
&= \partial_- \mathbf{X}_R \cdot \partial_- \mathbf{X}_R \\
&= l_s^2 \left(\sum_{m=-\infty}^{\infty} \alpha_m e^{-2im(\tau-\sigma)} \right) \cdot \left(\sum_{n=-\infty}^{\infty} \alpha_n e^{-2in(\tau-\sigma)} \right) \\
&= l_s^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \alpha_m \cdot \alpha_n e^{-2i(m+n)(\tau-\sigma)} \\
&= l_s^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \alpha_{(m-n)} \cdot \alpha_n e^{-2i((m-n)+n)(\tau-\sigma)} \\
&= 2l_s^2 \sum_{m=-\infty}^{\infty} \left(\frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n \right) e^{-2im(\tau-\sigma)}
\end{aligned}$$