p40 2.84 Virasoro generators の交換関係

力学変数をモード展開した成分の間のポアソン括弧

(2.51) $[\alpha_m^\mu, \alpha_n^\nu]_{P.B.} = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu]_{P.B.} = im\eta^{\mu\nu}\delta_{m+n,0}$

(2.52) $[\alpha_m^\mu, \tilde{\alpha}_n^\nu]_{P.B.} = 0$

と、Virasoro generators の定義 (2.74) (2.76)

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}\cdot\alpha_n\ ,\quad \tilde{L}_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \tilde{\alpha}_{m-n}\cdot\tilde{\alpha}_n$$

より、以下のポアッソン括弧[ ]は

$$[L_m, \alpha_k^\alpha]_{P.B.} = [\frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}^\mu \alpha_n^\mu, \alpha_k^\alpha]_{P.B.}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} [\alpha_{m-n}^\mu \alpha_n^\mu, \alpha_k^\alpha]_{P.B.}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \left([\alpha_{m-n}^\mu, \alpha_k^\alpha]_{P.B.}\alpha_n^\mu + \alpha_{m-n}^\mu[\alpha_n^\mu, \alpha_k^\alpha]_{P.B.}\right)$$

$$= \frac{i}{2}\sum_{n=-\infty}^{\infty} \left((m-n)\eta^{\mu\alpha}\delta_{m-n+k,0}\alpha_n^\mu + \alpha_{m-n}^\mu n\eta^{\mu\alpha}\delta_{n+k,0}\right)$$

$$= \frac{i}{2}\Big((m-(m+k))\alpha_{m+k}^\alpha + \alpha_{m+k}^\alpha(-k)\Big)$$

$$= -ik\alpha_{m+k}^\alpha$$

となるから、 Virasoro generators 間のポアッソン括弧[ ]は

$$[L_m, L_n]_{P.B.} = [L_m, \frac{1}{2}\sum_{k=-\infty}^{\infty} \alpha_{n-k}^\mu \alpha_k^\mu]_{P.B.}$$

$$= \frac{1}{2}\sum_{k=-\infty}^{\infty} [L_m, \alpha_{n-k}^\mu]_{P.B.}\alpha_k^\mu + \frac{1}{2}\sum_{k=-\infty}^{\infty} \alpha_{n-k}^\mu [L_m, \alpha_k^\mu]_{P.B.}$$

$$= \frac{i}{2}\sum_{k=-\infty}^{\infty} -(n-k)\alpha_{m+(n-k)}^\mu \alpha_k^\mu + \frac{1}{2}\sum_{k=-\infty}^{\infty} \alpha_{n-k}^\mu(-k)\alpha_{m+k}^\mu$$

$$= \frac{i}{2}\sum_{k=-\infty}^{\infty} -(n-k)\alpha_{m+n-k}^\mu \alpha_k^\mu + \frac{1}{2}\sum_{k=-\infty}^{\infty} \alpha_{n-k+m}^\mu(-k+m)\alpha_k^\mu$$

$$= \frac{i}{2}(m+n)\sum_{k=-\infty}^{\infty} \alpha_{m+n-k}^\mu \alpha_k^\mu$$

$$= i(m+n)L_{m+n}$$

同様にして、

$$[\tilde{L}_m, \tilde{L}_n]_{P.B.} = i(m+n)\tilde{L}_{m+n}$$

となる。

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