

$$[L_m, \alpha_k^\mu] = -k \alpha_{m+k}^\mu$$

$$[L_m, \tilde{\alpha}_k^\mu] = -k \tilde{\alpha}_{m+k}^\mu$$