$$
\begin{aligned}
L_{-1} &= \frac{1}{2}\sum_{k=-\infty}^{\infty} :\alpha_{-1-k}\cdot\alpha_k + \widetilde{\alpha}_{-1-k}\cdot\widetilde{\alpha}_k: \\
&= \sum_{k=0}^{\infty}(\alpha_{-1-k}\cdot\alpha_k + \widetilde{\alpha}_{-1-k}\cdot\widetilde{\alpha}_k)
\end{aligned}
$$