

$$\begin{aligned} L_2 &= \frac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{2-n} \cdot \alpha_n : \\ &= \sum_{n=2}^{\infty} \alpha_{2-n} \cdot \alpha_n + \frac{1}{2} \alpha_1^2 \end{aligned}$$